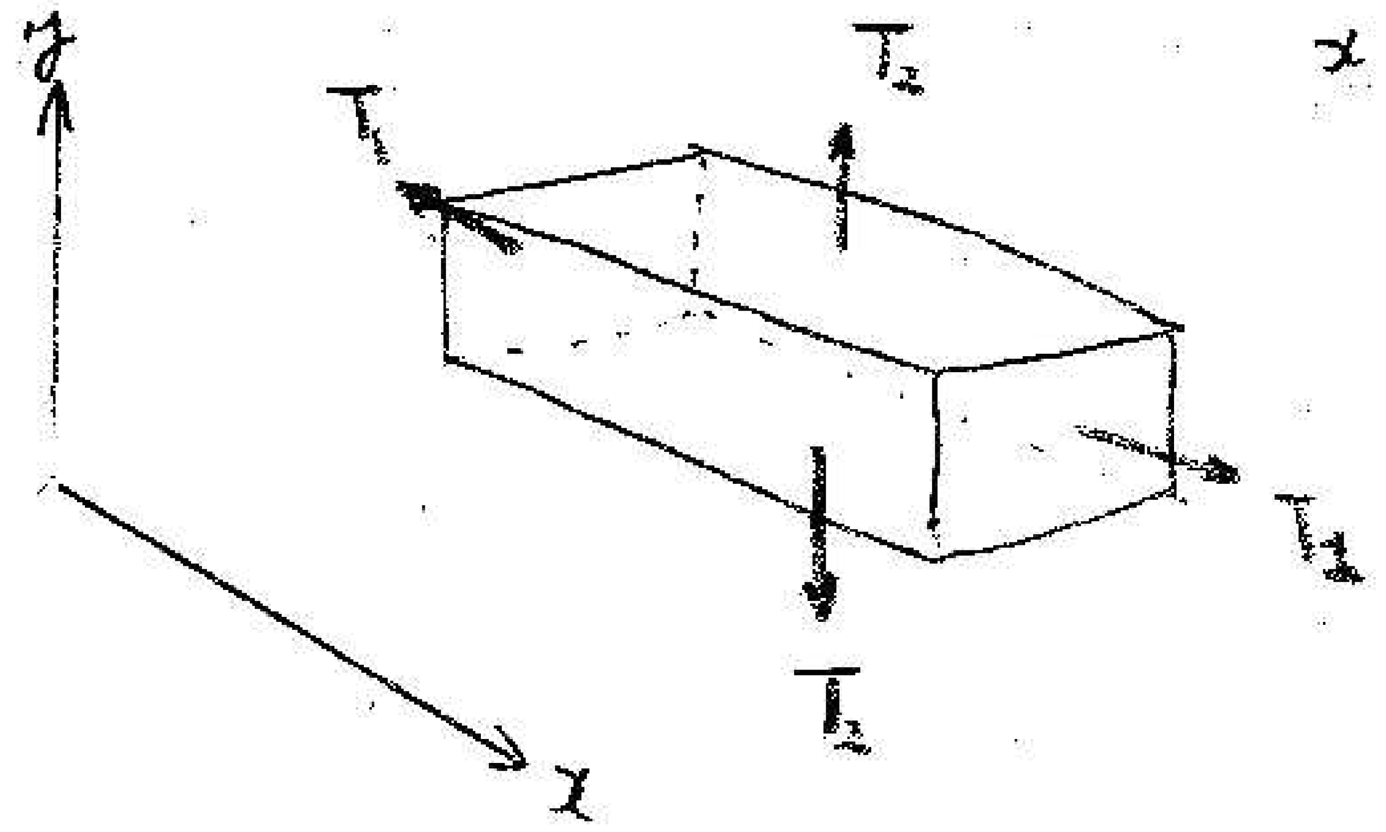
応力と歪の関係




x : direction y : direction

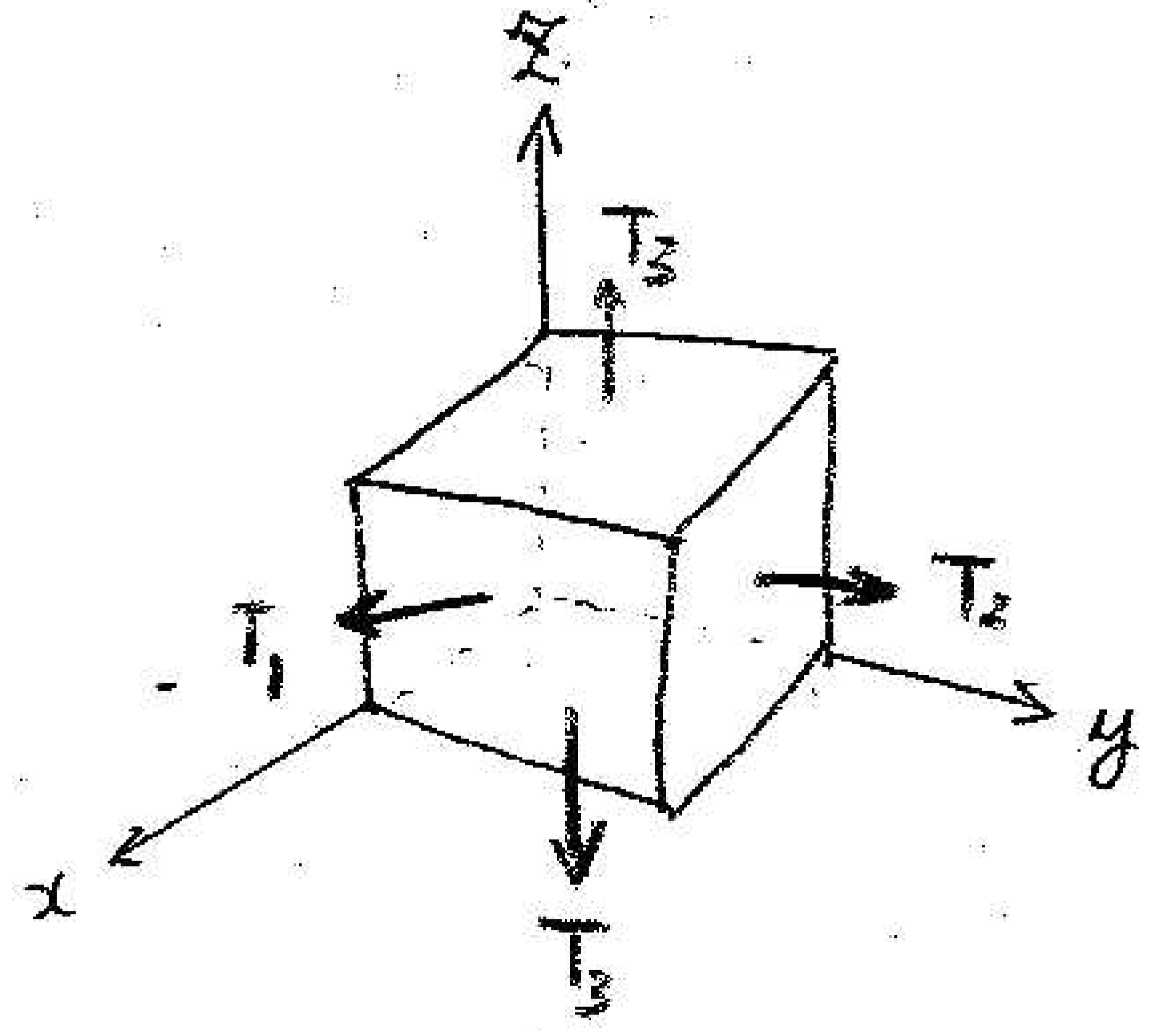
$$\varepsilon_1 = \frac{T_1}{E} \qquad \varepsilon_2 = -\sigma\frac{T_1}{E}$$

$$\varepsilon_2' = -\sigma\frac{T_2}{E}, \quad \varepsilon_1' = \frac{T_2}{E}$$

$$\varepsilon_x = \varepsilon_1 + \varepsilon_2 = \frac{1}{E}(T_1 - \sigma T_2) \quad (\equiv \varepsilon_1)$$

$$\varepsilon_y = \varepsilon_1' + \varepsilon_2 = \frac{1}{E}(T_2 - \sigma T_1) \quad (\equiv \varepsilon_2)$$

(主歪)



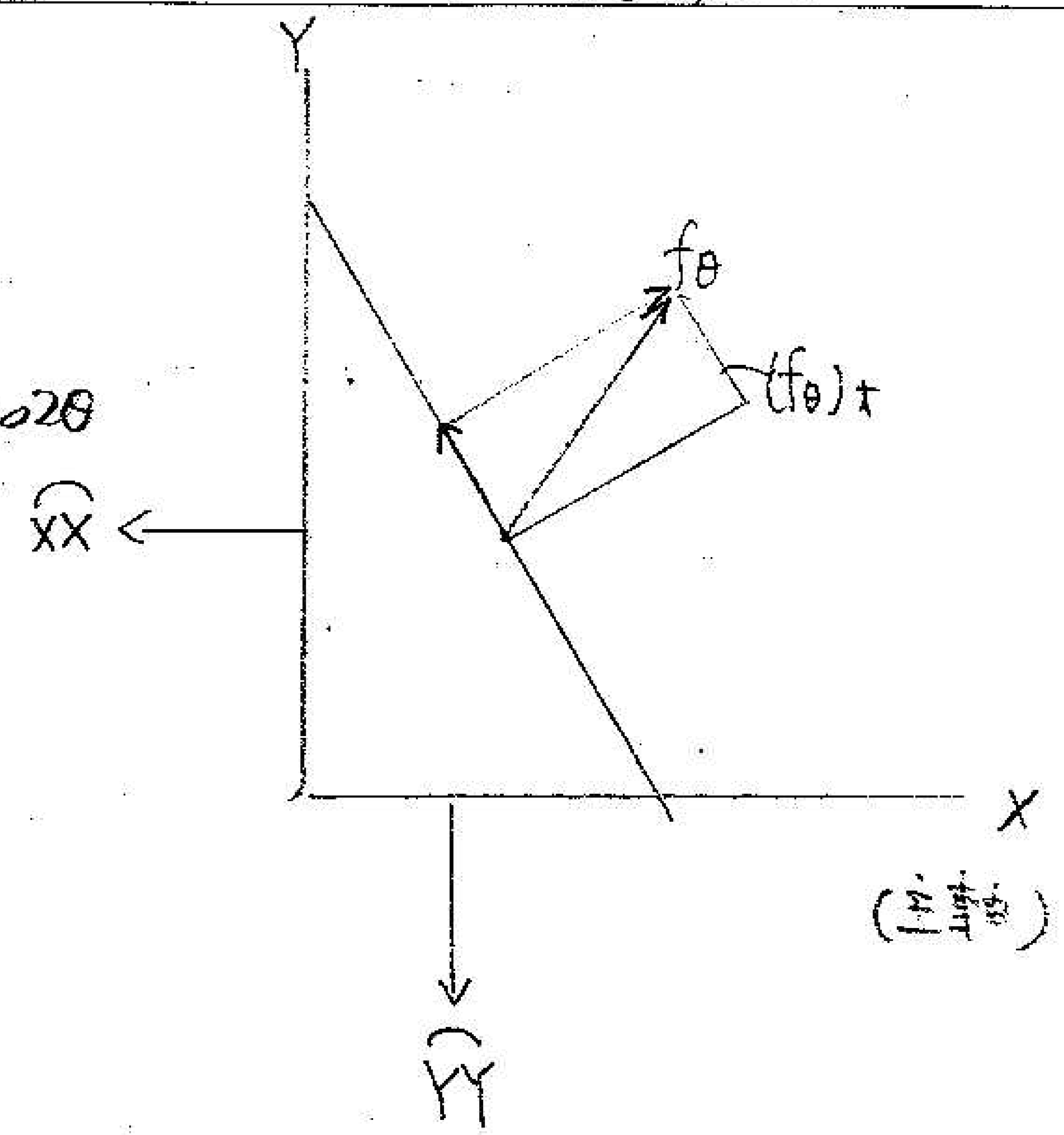

$$\left.\begin{aligned} \varepsilon_1 &= \frac{1}{E}\{T_1 - \sigma(T_2 + T_3)\} \\ \varepsilon_2 &= \frac{1}{E}\{T_2 - \sigma(T_3 + T_1)\} \\ \varepsilon_3 &= \frac{1}{E}\{T_3 - \sigma(T_1 + T_2)\} \end{aligned}\right\} \quad (2.89)$$

$$\left.\begin{aligned} T_1 &= \frac{\sigma E}{(1+\sigma)(1-2\sigma)}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \frac{E}{1+\sigma}\varepsilon_1 \\ T_2 &= \frac{\sigma E}{(1+\sigma)(1-2\sigma)}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \frac{E}{1+\sigma}\varepsilon_2 \\ T_3 &= \frac{\sigma E}{(1+\sigma)(1-2\sigma)}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \frac{E}{1+\sigma}\varepsilon_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} T_1 &= \lambda\Delta + 2G\varepsilon_1 \\ T_2 &= \lambda\Delta + 2G\varepsilon_2 \\ T_3 &= \lambda\Delta + 2G\varepsilon_3 \end{aligned}\right\} \qquad G = \frac{E}{2(1+\sigma)}$$

$$\lambda = \frac{\sigma E}{(1+\sigma)(1-2\sigma)} \qquad \Delta \equiv (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$

系　主平面には剪断力がない。




主軸を座標軸にとった場合には

(1.4.6)
$$\begin{cases} (f_\theta)_n = \frac{1}{2}(\widehat{XX}+\widehat{YY}) + \frac{1}{2}(\widehat{XX}-\widehat{YY})\cos 2\theta \\ (f_\theta)_t = \frac{1}{2}(\widehat{YY}-\widehat{XX})\sin 2\theta \end{cases}$$

剪断力が０であるのを主平面という。

$$\frac{1}{2}(\widehat{yy}-\widehat{xx})\sin 2\theta + \widehat{xy}\cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\widehat{xy}}{\widehat{xx}-\widehat{yy}} \quad (1.44)$$

$$\left.\begin{aligned} \theta &= \frac{1}{2}\tan^{-1}\frac{2\widehat{xy}}{\widehat{xx}-\widehat{yy}} \\ \theta &= \frac{1}{2}\tan^{-1}\frac{2\widehat{xy}}{\widehat{xx}-\widehat{yy}} + \frac{\pi}{2} \end{aligned}\right\} (1.4.5)$$

$$\frac{d(f_\theta)_n}{d\theta} = -(\widehat{xx}-\widehat{yy})\sin 2\theta + 2\widehat{xy}\cos 2\theta = 0$$

$$(\widehat{yy}-\widehat{xx})\sin 2\theta + 2\widehat{xy}\cos 2\theta = 0$$

$$(\widehat{yy}-\widehat{xx})\tan 2\theta + 2\widehat{xy} = 0$$

$$\tan 2\theta = \frac{2\widehat{xy}}{\widehat{xx}-\widehat{yy}} \quad (1.44')$$

$$\therefore\ \theta = \frac{1}{2}\tan^{-1}\frac{2x}{\widehat{xx}-\widehat{yy}}$$

$$\theta = \frac{1}{2}\tan^{-1}\frac{2x}{\widehat{xx}-\widehat{yy}} + \frac{\pi}{2}$$

S.45.6.8

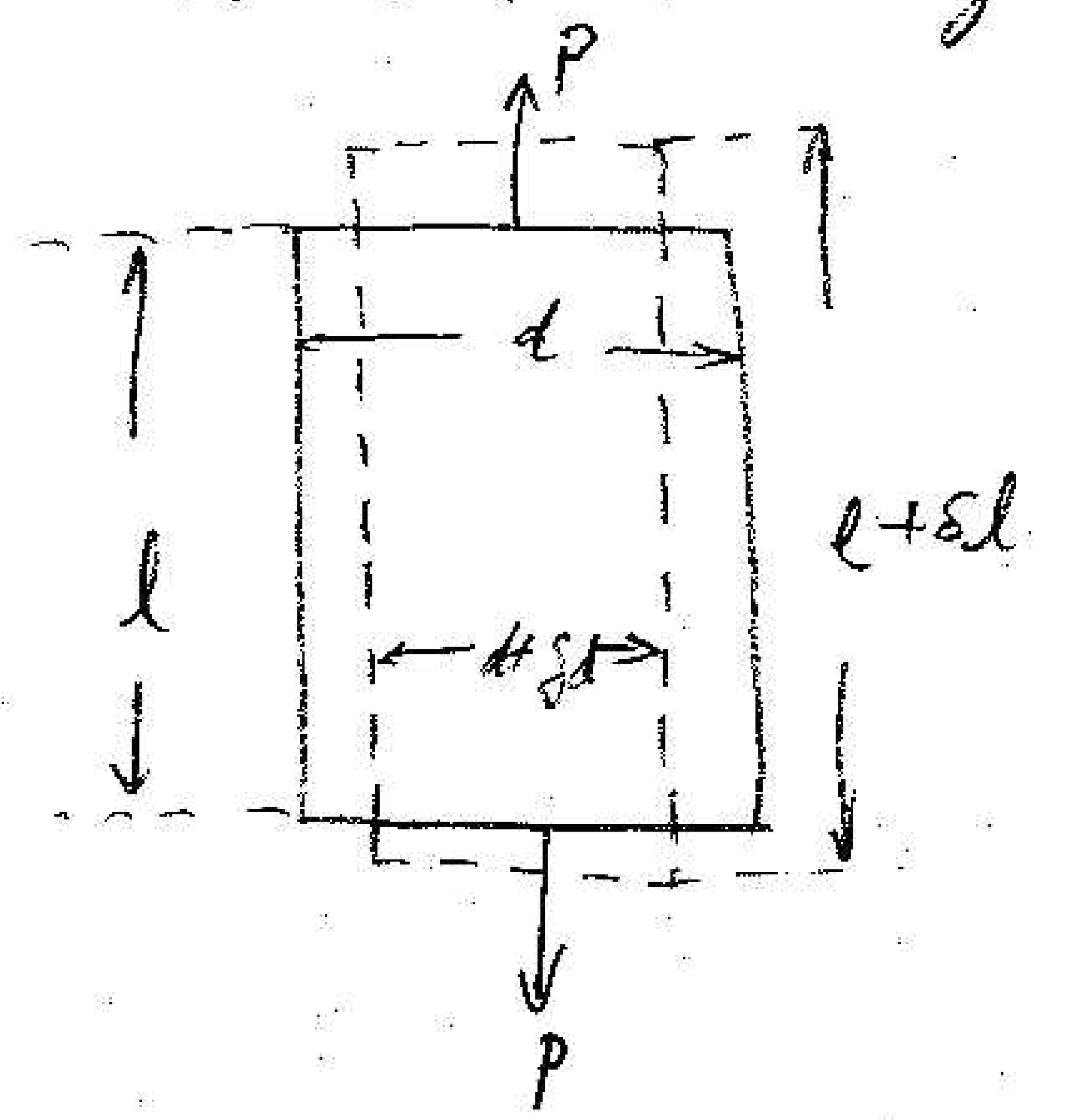
## 2.4 歪 (strain)

a) 引張歪 } 垂直歪 (normal strain)

b) 圧張歪

c) 剪断歪

A) 縦歪 (longitudinal strain)




$$\varepsilon_1 = \frac{\delta l}{l} \quad (2.4.1)$$

$f_\theta$ = kg/mm²

kg/cm²

引張荷重の場合は正号

圧縮荷重の場合は負号

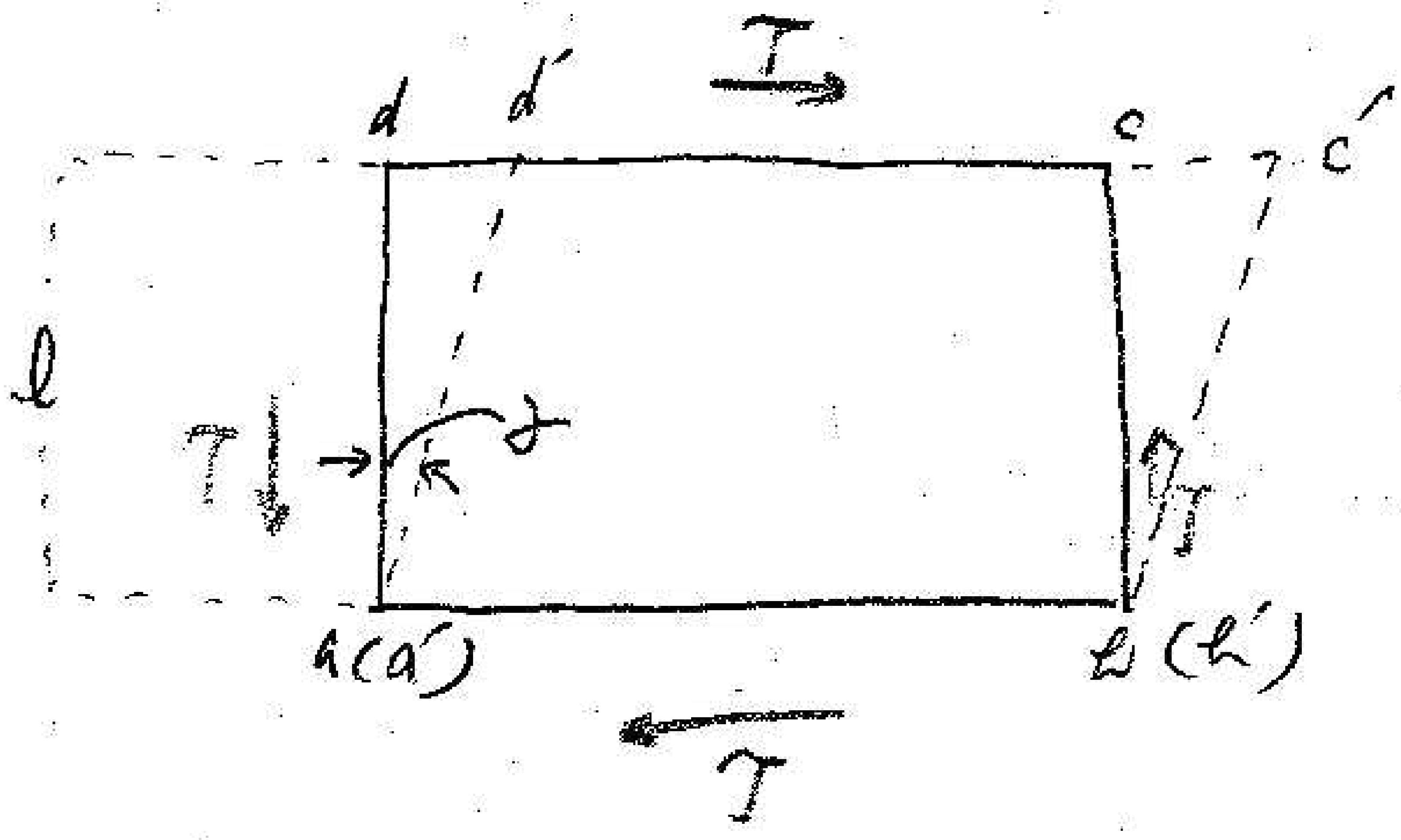
B) 横歪 lateral strain

$$\varepsilon_2 = \frac{\delta d}{d}$$

引張荷重の場合は負号

圧縮荷重の場合は正号

C) 剪断歪 shear strain




$$\frac{dd'}{ad} = \frac{\delta l}{l} = \tan\gamma \fallingdotseq \gamma \quad (2.4.3)$$

D) 体積歪 (Bulk strain)

$$\varepsilon_v = \frac{\Delta}{V} \left( = \frac{\text{change of}}{\quad} \right)$$

2.5. 弾性係数 (Elastic modulus)

弾性係数 (young's modulus)
ヤング率

$$E = \frac{f_0}{\varepsilon_1} \qquad \left( f_0 = \frac{P}{A} \right) \quad (2.4.5)$$

$$\left( \frac{\text{normal stress}}{\text{normal strain}} \right)$$

steel 22,000 ~ 20,000 kg/mm²

cast Iron 10,000 ~ 8,000 kg/mm²

$$\frac{P/A}{\delta l / l}$$

B) ポアソン比 (Poisson's Ratio)

$$\sigma = -\frac{\varepsilon_2}{\varepsilon_1} \quad (2.4.6)$$

$$m = \frac{1}{\sigma} \quad \text{(ポアソン数)}$$

(Poisson's Number)

工業用金属 = 0.25 ~ 0.35 mild steel $\sigma = 0.26$

ゴム = 0.5

コンクリート = 0.12 以下 copper, Brass $\sigma = 0.33$